CAD Government Bond Bootstrapping Procedure

An algorithm is presented for bootstrapping a discount factor curve. The bootstrapping procedure uses an input set of instruments with different maturities (i.e., Canadian government money market securities and bonds) to generate successive points on a discount factor curve.

The Canadian zero curves generated will be used to generate particular risk measures, for example DV01's. Moreover, the zero rate curves are not intended for use in pricing (P&L) applications (ref https://finpricing.com/lib/IrInflationCurve.html).

The Government Bond Bootstrapping interface requires as input a set of financial instruments of the type below, sorted by order of increasing time to maturity:

- Short term money market instruments (i.e., CAD T-Bills with maturity not more than one year),
- Medium to long term "on the run" Canadian government bonds,
- Medium to long term "off the run" Canadian government bonds.

To determine discount factors at times intermediate to control points, we apply a particular interpolation technique. There are three available:

- LINEAR

- LOG_LINEAR
- TIME_WEIGHTED_LINEAR

The LINEAR scheme interpolates zero rates linearly between successive control points on the zero curve; that is, if $r(t_1)$ and $r(t_2)$ are bootstrapped continuously compounded zero rates at successive control points, then

$$r(t) = r(t_1) + \left(r(t_2) - r(t_1)\right)\frac{t - t_1}{t_2 - t_1} \text{ where } t_1 \le t \le t_2. \qquad (1)$$

The LOG_LINEAR scheme interpolates linearly between $\ln r(t_1)$ and $\ln r(t_2)$, that is,

$$\ln r(t) = \ln r(t_1) + \left(\ln r(t_2) - \ln r(t_1)\right)\frac{t - t_1}{t_2 - t_1} \quad \text{or}$$

$$r(t) = r(t_1)\left(\frac{r(t_2)}{r(t_1)}\right)^{\left(\frac{t-t_1}{t_2-t_1}\right)}. \qquad (2)$$

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he TIME_WEIGHTED_LINEAR scheme interpolates between $r(t_1)t_1$ and $r(t_2)t_2$,

$$r(t) = \frac{1}{t}\left(r(t_1)t_1 + (r(t_2)t_2 - r(t_1)t_1)\frac{t - t_1}{t_2 - t_1}\right), \qquad (3)$$

where $t > 0$ and $t_1 \le t \le t_2$. Since $t_2 > t_1$, the TIME_WEIGHTED_LINEAR scheme weights rate information farther in the future more heavily than rate information in the near future.

The CAD Government Bond Bootstrapping proceeds in two phases. The first phase uses short term instruments maturing in the near future; here discount factors are directly inferred from the prices of these instruments, which typically mature in one year or less. Consider, for example, a CAD government money market instrument with

- D calendar days between settlement date, S, and maturity date, $S+D$, and
- with a corresponding simple forward interest rate r.

Then

$$ {}_{S}df_{S+D} = \frac{1}{1 + r\frac{D}{365}}, \tag{4} $$

where the "365" factor arises from the ACT/365 market convention used for Canadian money market instruments, is the forward price at the date, S, of a zero coupon bond with maturity date, $S+D$, and unit face value.

Canadian Government Bond yield to maturity (YTM), pricing and accrued interest conventions are unusual and are given in Appendix 1. Note that standard SIA bond yield pricing formulations differ from Canadian Government Bond pricing formulations.

Phase one uses price quote information for four money market instruments. We compute

$$ {}_{0}df_{1} = \frac{1}{1 + 0.02775\frac{1}{365}} = 0.999923978382465. $$

The continuously compounded discount rate for a single day is then

$$r_1 = \frac{-\ln\left({}_0 df_1\right)}{1/365}.$$

Phase two uses eight Canadian government bonds. We compute the settlement dirty price (at T+2, i.e., 4 Nov 2002), as a function of the forward YTM,

$$P_d = 103.77087553.$$

Then

$$\left({}_0 df_4\right) P_d = 103.740406 ,$$

is the bond's dirty price discounted to the trade date.

Let a bond have the cash flow, c_i, at date, t_i, for $i = 1,\ldots,n$. Our Benchmark applies a Newton iterative scheme to solve

$$\left({}_0 df_S\right) P_d = \sum_{i=1}^{n} c_i \left({}_0 df_{t_i}\right),$$

for the unknown zero rate at t_n where S is the settlement date. We use the "Time Weighted Linear" interpolation scheme to determine intermediate discount rates.