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$$= \frac{4}{3a} \int_{x_2}^{x_1} \frac{x(2a^2+ax)^{\frac{3}{2}}}{(a+x)\sqrt{\{4ac^2(a+x)-x^2\}}} dx$$

$$= \frac{4a}{3} \int_{x_2}^{x_1} \frac{x\sqrt{\{2a^2+ax\}}dx}{(a+x)\sqrt{\{4ac^2(a+x)-x^2\}}} + \frac{4}{3} \int_{x_2}^{x_1} \frac{x\sqrt{\{2a^2+ax\}}dx}{\sqrt{\{4ac^2(a+x)-x^2\}}}.$$

Let $x=2ac^2+2ac\sqrt{\{1+c^2\}}\cos 2\theta$, $1+c^2+c\sqrt{\{1+c^2\}}=b^2$, $2c\sqrt{\{1+c^2\}}/b^2=e^2$, $4c\sqrt{\{1+c^2\}}/\{2b^2-1\}=d$.

$$\therefore S_p = \frac{16a^2b\sqrt{2(b^2-1)}}{3(2b^2-1)} \int_0^{\frac{1}{2}\pi} \frac{d\theta}{(1-d\sin^2\theta)\sqrt{\{1-e^2\sin^2\theta\}}}$$

$$- \frac{16a^2b\sqrt{2(2c\sqrt{\{1+c^2\}}+b^2e^2-e^2)}}{3(2b^2-1)} \int_0^{\frac{1}{2}\pi} \frac{\sin^2\theta d\theta}{(1-d\sin^2\theta)\sqrt{\{1-e^2\sin^2\theta\}}}$$

$$+ \frac{32a^2bce^2\sqrt{\{2(1+c^2)\}}}{3(2b^2-1)} \int_0^{\frac{1}{2}\pi} \frac{\sin^4\theta d\theta}{(1-d\sin^2\theta)\sqrt{\{1-e^2\sin^2\theta\}}}$$

$$+ \frac{1}{3} a^2 b \sqrt{2(b^2-1)} \int_0^{\frac{1}{2}\pi} \sqrt{\{1-e^2\sin^2\theta\}} d\theta$$

$$- \frac{2}{3} a^2 b c \sqrt{\{2(1+c^2)\}} \int_0^{\frac{1}{2}\pi} \sqrt{\{1-e^2\sin^2\theta\}} \sin^2\theta d\theta.$$

$$\therefore S_p = A\Pi(e, -d, \frac{1}{2}\pi) + \frac{B}{d}\{F(e, \frac{1}{2}\pi) - \Pi(e, -d, \frac{1}{2}\pi)\}$$

$$+ \frac{C}{d^2e^2}\{e^2\Pi(e, -d, \frac{1}{2}\pi) + dE(e, \frac{1}{2}\pi) - (d+e^2)F(e, \frac{1}{2}\pi)\} + DE(e, \frac{1}{2}\pi)$$

$$+ \frac{E}{3e^2}\{1-2e^2\}E(e, \frac{1}{2}\pi) - (1-e^2)F(e, \frac{1}{2}\pi).$$

$$\therefore S = 4\pi a^2 c(1+c^2) + (A - \frac{B}{d} + \frac{C}{d^2})\Pi(e, -d, \frac{1}{2}\pi) + (\frac{C}{de^2} + D$$

$$+ \frac{E(1-2e^2)}{3e^2})E(e, \frac{1}{2}\pi) + (\frac{B}{d} - \frac{C(d+e^2)}{d^2e^2} - \frac{E(1-e^2)}{3e^2})F(e, \frac{1}{2}\pi).$$

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MECHANICS.
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139. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

A homogeneous sphere, radius $r=50$ inches, makes $m=30$ revolutions around an axis every second. The mass begins to disappear from the surface into space at a rate exactly sufficient to cause the diameter to decrease uniformly at the rate of $(1/n)$ th= $1/1000$ th of

a linear inch per second. At what rate per second is the angular velocity of the sphere changing the instant the diameter becomes $p=10$ inches? What is the diameter of the sphere when the rate of disappearance of matter is midway between minimum and maximum? When is the angular velocity a maximum? How does this maximum angular velocity compare with the original angular velocity? What is the diameter of the sphere when the paracentric force is (1) a maximum, and (2) a minimum?

No solution of this problem has been received.

140. Proposed by J. F. LAWRENCE, A. B., St. Louis. Mo.

A long row of particles, each mass m , is placed on a smooth horizontal table. Each is connected with the two adjacent ones by similar light elastic strings of natural length l . They receive small longitudinal disturbances such that each of them proceeds to perform a harmonic oscillation. Prove that there will be two waves of vibration in opposite directions with the same velocity, viz, $l' \sqrt{\frac{E}{ml} \frac{q}{\pi} \sin \frac{\pi}{q}}$, when l' is the average distance between two successive particles, q the number of intervals between two particles in the same phase, and E the modulus of elasticity. [*Mathematical Tripos*, 1873.]

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $D=d/dt$, and the equation of motion is $y_{k+1}-2y_k+y_{k-1}=\frac{D^2}{c^2}y_k$ where $c^2=E/(ml)$. To solve this equation of differences, treat D as constant and put $y_k=Cx^k$ where C and x are two constants. Substituting and reducing, $x-2+1/x=(D/c)^2$.

$$\therefore \sqrt{x-1}/\sqrt{x}=\pm D/c. \quad \sqrt{x+1}/\sqrt{x}=\pm \sqrt{\{4+(D/c)^2\}}=\pm 2\sqrt{1+\left(\frac{D}{2c}\right)^2}$$

$$\therefore \sqrt{x}=\sqrt{1+\left(\frac{D}{2c}\right)^2}-\frac{D}{2c}=E.$$

$$\therefore y_k=E^{2k}f(t)+E^{-2k}F(t).$$

Let us express $f(t)$ and $F(t)$ in a series whose general term is $A\cos(2c\sin\theta t+\omega)$.

The operator E under the radical contains only even powers of D and we can write $-(2c\sin\theta)^2$ for D^2 .

$$\therefore E\cos(2c\sin\theta t+\omega)=\cos(2c\sin\theta t+\omega-\theta).$$

$$\therefore E^{2k}\cos(2c\sin\theta t+\omega)=\cos(2c\sin\theta t+\omega-2k\theta).$$

$$E^{-2k}\cos(2c\sin\theta t+\omega)=\cos(2c\sin\theta t+\omega+2k\theta).$$

$$\therefore y_k=\Sigma A\cos(2c\sin\theta t+\omega-2k\theta)+\Sigma B\cos(2c\sin\theta t+\omega+2k\theta).$$

If we substitute in any one term of the first series $k+1$ for k and $t+T$ for t , where $T=\frac{\theta}{c\sin\theta}$, the term is unaltered..